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STABLE RANK IN HOLOMORPHIC FUNCTION ALGEBRAS

BY

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Introduction

The concept of the stable rank of a ring, introduced by H. Bass [1], has been very useful in treating some problems in algebraic K-theory. In this paper we show how this concept is related to the structure of a commutative Banach algebra A. For example, we show that A has a finite stable rank if its spectrum $X(A)$ has finite dimension.

First, we prove, in an analytical way, that the algebra of continuous functions on the disc $\bar{\Delta} = \{z \in \mathbf{C}\colon |z| \leq 1\}$ which are holomorphic on its interior Δ, has stable rank 1. In Sections 2 and 3 we give other proofs of this fact, but it is convenient to have a classical proof. Furthermore, some ideas contained in it lead to the notion of punctual stability, to be developed and applied in Section 3.

In Section 2 we mention some results, taken from [4], and we apply them to the study of the stable rank of some algebras of holomorphic functions.

In Section 3 we introduce the concept of stability of a ring A at a point $g \in A$. In the case of a commutative Banach algebra A we relate this concept to the topological structure of some subsets of A^n. As an application we prove that $\mathscr{P}(X)$ and $\mathscr{R}(X)$ (see definitions below) have stable rank one. Finally we give a list of some open problems.

Most results about Banach algebras we prove here may be stated for topological algebras A whose unit groups $A^{\cdot}$ are open and the inversion is a homeomorphism of $A^{\cdot}$. The proofs are analogous to those given here.

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Section 1

This section considers notations and preliminary results. In this paper, rings and algebras have identity. The group of units of A is denoted by $A^{\cdot}$. Given a ring A, $a \in A^n$ is *unimodular* if there exists $b \in A^n$ such that $\langle b, a\rangle = \sum_{i=1}^{n} b_i a_i = 1$. We denote by $U_n(A)$ the unimodular elements of A^n. We say

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