ILLINOIS JOURNAL OF MATHEMATICS
Volume 42, Number 2, Summer 1998

AUTOMORPHISMS OF METABELIAN GROUPS WITH TRIVIAL CENTER

RÜDIGER GÖBEL AND AGNES T. PARAS

1. Introduction

Let $F(n)$ denote the free group of rank n and let $B(n) = F(n)/F(n)''$, the free metabelian group of rank n. The automorphism group $\mathrm{Aut}(B(n))$ has been independently and jointly investigated by Bachmuth and Mochizuki in a series of papers dating from 1965 to 1987. In [1], the outer automorphism group $\mathrm{Out}(B(2))$ is shown to be isomorphic to $GL_2(\mathbb{Z})$. When $n = 3$, $\mathrm{Aut}(B(n))$ has been shown to be infinitely generated in [2]. For $n \geq 4$, they showed [3] that

$$\mathrm{Aut}(F(n)) \to \mathrm{Aut}(B(n)) \to 1;$$

i.e., every automorphism of $B(n)$ is induced by an automorphism of $F(n)$ and hence, $\mathrm{Aut}(B(n))$ is finitely generated. This is carried out using the faithful Magnus representation of $IA(B(n))$ as a subgroup of $GL_n(\mathbb{Z}[F(n)/F(n)'])$ ($IA(G)$ is the normal subgroup of $\mathrm{Aut}(G)$ consisting of automorphisms of G which induce the identity on the quotient G/G'), and ideas and methods influenced by matrices and matrix groups over integral Laurent polynomial rings.

Instead of considering the automorphism group of a given metabelian group, we propose to approach the problem from the opposite direction, namely:

> Which groups can be realized as the automorphism groups of metabelian groups?

That is, for which groups H does there exist a metabelian group G such that $\mathrm{Aut}\, G$ is isomorphic to H?

The case when G is a torsion free, nilpotent group of class 2, hence metabelian with non-trivial center, has been considered by Dugas and Göbel. In [8], they adapt Zalesskii's matrix construction of a torsion free, nilpotent group of rank 3 and class 2 with no outer automorphisms, to show that any group H can be realized as

$$\mathrm{Aut}(G)/\mathrm{Stab}(G) \cong H$$

Received May 11, 1997.
1991 Mathematics Subject Classification. Primary 08A35,20C07, 20F29; Secondary 04A20, 20K20.
This work was done at the University of Essen, where the first author was supported by the German-Israeli Foundation for Scientific Research and Development and the second author was supported by the German Academic Exchange Servie (DAAD).