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INTEGRAL REPRESENTATIONS FOR POSITIVE SOLUTIONS OF THE HEAT EQUATION ON SOME UNBOUNDED DOMAINS

BY

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0. Introduction

The main result (Theorem 3.4) in this paper extends the integral representation theorem of Widder [7, Theorem 5.2, p. 143] for positive solutions of the heat equation on $\mathbf{R}_+ \times (0, T)$ to positive solutions on $\mathbf{R}^{n-1} \times \mathbf{R}_+ \times (0, T)$ for arbitrary n. (This result may be part of the mathematical folk-lore but no proof exists in the literature.) Then in Section 4, the Appell transform is used to obtain similar results on $\mathbf{R}^{n-1} \times \mathbf{R}_+ \times (-\infty, 0)$ and $\mathbf{R}^{n-1} \times \mathbf{R}_+ \times \mathbf{R}$, thus generalizing the result in [4, Theorem A] which was obtained by different methods. The integral representation obtained for positive solutions on $\mathbf{R}^{n-1} \times \mathbf{R}_+ \times \mathbf{R}$ verifies an assumption needed in [6, Remarks (2)] to compare fine limits with parabolic limits at points on $\mathbf{R}^{n-1} \times \{0\} \times \mathbf{R}$.

1. Preliminaries

Let $0 < T \leq \infty$,

$$X = \mathbf{R}^{n-1} \times \mathbf{R}_+ \times (0, T) = \{(x', x_n, t) : x' \in \mathbf{R}^{n-1}, x_n > 0, 0 < t < T\},$$

$$H = \mathbf{R}^{n-1} \times \mathbf{R}_+ \times \{0\} \text{ (the horizontal boundary of } X),$$

$$V = \mathbf{R}^{n-1} \times \{0\} \times [0, T) \text{ (the vertical boundary of } X),$$

$$B = H \cup V, \text{ and } V_+ = V \setminus (\mathbf{R}^{n-1} \times \{0\} \times \{0\}).$$

The fundamental solution of the heat equation $\Delta_x u = \partial u / \partial t$ on $\mathbf{R}^n \times \mathbf{R}$ is given by

$$W(x, t; y, s) = \begin{cases} [4\pi(t - s)]^{-n/2} \exp\left(- \dfrac{\|x - y\|^2}{4(t - s)} \right) & \text{if } t > s \\ 0 & \text{if } t \leq s. \end{cases}$$

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