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A SIMPLIFICATION OF ROSAY'S THEOREM ON GLOBAL SOLVABILITY OF TANGENTIAL CAUCHY-RIEMANN EQUATIONS

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In a recent paper by Rosay [6], the global solvability of the tangential Cauchy-Riemann complex $\bar{\partial}_b$ on the boundaries of weakly pseudo-convex domains is studied. He proved the following theorem:

THEOREM 1. *Let Ω be a weakly pseudo-convex domain in C^n with smooth boundary $b\Omega$. Assuming $n \geqq 2$ and $p \leqq n$, the equations*

$$(1) \qquad \bar{\partial}_b u = \alpha, \quad \text{where } \alpha \text{ is a smooth } (p, n-1) \text{ form on } b\Omega,$$

has a smooth solution u if and only if α satisfies

$$(2) \qquad \int_{b\Omega} \alpha \wedge \Phi = 0 \quad \text{for every } \bar{\partial}\text{-closed } (n-p, 0) \text{ form } \phi.$$

The same result for *strictly* pseudo-convex domains has been proved by Henkin in [2] using the integral representation for the $\bar{\partial}$ operator.

In his paper, Rosay also noted parenthetically that, following the work of Kohn and Rossi [5] and Kohn [3], the necessary and sufficient conditions for the solvability of the equations

$$(3) \quad \bar{\partial}_b u = \alpha, \quad \text{where } \alpha \text{ is a smooth } (p, q) \text{ form on } b\Omega \text{ and } q < n - 1,$$

are

$$(4) \qquad \bar{\partial}_b \alpha = 0$$

Rosay's method for proving Theorem 1 is to use the solution of the $\bar{\partial}$-Neumann problem in an ingenious way. However, it is not the most direct one. In this note we shall show that, with a simple argument of integration by parts, Kohn and Rossi's method can be directly extended to $(p, n-1)$ forms, thus providing a unified approach to the solvability of equations (1) and (3).

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