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ON BROWNIAN MOTION WITH IRREGULAR DRIFT

BY

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Dedicated to the memory of Theophil Henry Hildebrandt

1. Introduction

Our objective is to describe some class of diffusion processes on the line generated by Feller's generalized second order differential operator $D_m D_p^+$ as strong solutions of stochastic equations. In contrast to earlier papers [6]–[8], which investigated the case of nonsmooth speed measure m, we are now concerned with irregularities of the natural scale p.

For a classical second order differential operator

$$\frac{1}{2}\frac{d^2}{dx^2} + b\frac{d}{dx}$$

with unit diffusion coefficient and a locally square integrable drift coefficient b satisfying some growth condition at the boundaries, the situation is well known. Here the scale functions p_b and m_b are defined by the formulae

$$dp_b(x) = \exp\left\{-2\int_0^x b(y)\,dy\right\}dx,$$

$$dm_b(x) = \exp\left\{2\int_0^x b(y)\,dy\right\}2\,dx, \quad x \in \mathbf{R};$$

cf. the paper of Orey [14]. We observe that the derivative π_b of the natural scale p_b uniquely solves the linear integral equation

$$\pi_b(x) = 1 - 2\int_0^x \pi_b(y)b(y)\,dy, \quad x \in \mathbf{R}.$$

For a given standard Brownian motion W, to each initial state $x \in \mathbf{R}$ there

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