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THE RIEMANNIAN OBSTACLE PROBLEM

BY

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1. Introduction

In this paper we consider the local distance geometry of Riemannian manifolds with boundaries. We view a boundary component as an obstacle around which a geodesic can bend, or at which a geodesic can end. Our emphasis is on the structure of fields of geodesics. In the presence of an obstacle, the description of such fields of geodesics in terms of differential equations is no longer feasible; as an alternative, we produce a key differential inequality which functions as a one-sided version of the Jacobi equation. In consequence we obtain local bipoint uniqueness and a geometric estimate on the cut distance, that is, the distance below which bipoint uniqueness holds. On the other hand, unique determination of geodesics by their initial tangents (Cauchy uniqueness) clearly fails; we have developed basic techniques to establish properties of the field of geodesics with a common tangent.

It is immediately clear that the Riemannian obstacle problem is natural from the variational and mechanical points of view. Consider, for example, a string in Euclidean 3-space, stretched around an obstacle, sometimes following the obstacle, sometimes travelling through the air. Or, the problem can be stated as that of analyzing the propagation of wavefronts (level surfaces of the distance function) around an obstacle. If the wavefronts are due to a point source of light or sound in a nonhomogeneous medium, then the appropriate geometry is Riemannian rather than Euclidean. Arnol'd has considered the obstacle problem in a series of recent papers. Arnol'd is carrying out a general program which identifies standard singularities related to the geometry of groups generated by reflections with normal forms for singularities occurring in variational problems. This investigation leads him to variational problems with one-sided constraints, and in particular to an analysis of the singularities of wavefronts for Euclidean obstacles in general position. (For a survey, see [5]; see also [6], [7], [8].) Arnol'd achieves an analysis in this case even though, as he states, "the problem of going around an obstacle has not yet been solved even in Euclidean 3-space" [5].

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