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ON A FORMULA FOR ALMOST-EVEN ARITHMETICAL FUNCTIONS

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Introduction

For an arithmetical function the property of being almost-even is a special case of limit-periodicity, which is itself a special case of almost-periodicity.

1.1. An arithmetical function f is said to be *almost-periodic-B* (more precisely almost-periodic-B^1) if, given $\varepsilon > 0$, there exists a trigonometric polynomial P,

$$P(n) = \sum_{k=1}^{m} \lambda_k e(\alpha_k n), \quad \text{where } e(t) = \exp(2\pi i t),$$

such that

$$\limsup_{x \to \infty} \frac{1}{x} \sum_{n \le x} |P(n) - f(n)| \le \varepsilon. \tag{1}$$

This implies that $\sum_{n \le x} |f(n)| = O(x)$ and that, for each real α,

$$\lim_{x \to \infty} \frac{1}{x} \sum_{n \le x} f(n) e(-\alpha n) \text{ exists, say is } C(f, \alpha).$$

The spectrum of f is the (at most denumerable) subset $\operatorname{Sp} f$ of the quotient group $\mathbf{R}/\mathbf{Z}$ consisting of the residue-classes modulo 1 of those α for which $C(f, \alpha) \neq 0$.

The Fourier series of f is the formal sum $\sum C(f, \alpha) e(\alpha n)$ extended to those $\alpha \in [0, 1[$ whose residue-class modulo 1 belongs to $\operatorname{Sp} f$.

The arithmetical function f is said to be *limit-periodic-B* if, given $\varepsilon > 0$, there exists a *periodic* arithmetical function P such that (1) holds.

Since a periodic arithmetical function can be expressed by a trigonometric polynomial, this implies that f is almost-periodic-B. Its spectrum is contained in $\mathbf{Q}/\mathbf{Z}$ (i.e., $C(f, \alpha) = 0$ when α is irrational).

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