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GROUPS AND CENTRAL ALGEBRAS

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If K is a field and A is a finite dimensional central simple K-algebra, then the Brauer class of A contains a crossed product (cf. [4, page 379], [10, page 474]). The algebra of real quaternions is a twisted group algebra of the Klein-4-group over the field of real numbers $\mathbf{R}$; it is also the algebra obtained from the group algebra $\mathbf{R}[G]$, where G is the group of quaternions, by identifying the center of G with $\{1, -1\}$ of $\mathbf{R}$. The similar construction for the dihedral group of order 8 gives the algebra of 2×2-matrices over $\mathbf{R}$. It turns out that twisted group algebras are the algebras obtained from group algebras by identifying a central subgroup with a subgroup of the field's multiplicative group. We also determine when the algebras obtained from group algebras by such identifications are central.

A group G is called completely central if for every non-central $g \in G$ with only finitely many conjugates, there is a central $1 \neq n \in G$ such that g is a conjugate of ng. The class of completely central groups contains all free groups, all nilpotent class 2-groups, all torsion free nilpotent groups and all groups of central type. However, there are nilpotent class 3-groups that are not completely central groups (e.g., the dihedral group of order 16) and there are nilpotent class 2-groups that are not of central type (e.g., one of the groups of order 64). We characterize groups of central type in the class of finite completely central groups.

K will always denote a non-trivial commutative ring with 1. Let $K^{\times}$ denote the group of units of K. The center of a group G will be denoted by $\zeta(G)$ and the center of an algebra A will be denoted by $\zeta(A)$. The conjugacy class of $g \in G$ will be denoted by $\mathrm{Cl}(g)$.

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1. Suppose K is a commutative ring with 1, G is a group, N is a central subgroup of G and α is a homomorphism of N into $K^{\times}$. The algebra obtained from the group algebra $K[G]$ by identifying n with $\alpha(n)$ for every $n \in N$ will be denoted by $KG\alpha$. The ideal of $K[G]$ generated by $\{n - \alpha(n)1 \mid n \in N\}$ will

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