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FITTING CLASSES OF $\mathscr{S}_1$-GROUPS, I

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1. Introduction

Since they were first introduced by Fischer [1], Fitting classes and the associated injectors of a finite soluble group have provided one of the most fruitful topics in the theory of finite soluble groups; a useful introduction to the subject is contained in the essay by Hawkes [4]. However, there have been few attempts to obtain a similar theory in infinite groups, particularly in the case of groups which are not locally finite and in which, therefore, one has little hope of using any form of Sylow theory. One positive result that has been obtained is the existence and conjugacy of nilpotent (hypercentral) injectors in polycyclic groups ($\mathscr{S}_1$-groups) [8]. Our aim here is to give a general theory of Fitting classes of $\mathscr{S}_1$-groups and investigate the extent to which this will give rise to the existence and conjugacy of injectors.

An $\mathscr{S}_1$-group is one possessing a finite normal series in which the factors are abelian groups of finite rank whose torsion subgroups are Černikov groups. Our terminology is taken from Robinson [7, Part 2, p. 137]; $\mathscr{S}_1$-groups have also been called *soluble groups of type* A_3 by Mal'cev [6]. One of the properties of $\mathscr{S}_1$-groups that will be of crucial importance throughout is that they are nilpotent-by-abelian-by-finite [7, Theorem 3.25].

Throughout we shall work within a subclass $\mathscr{X}$ of $\mathscr{S}_1$ which contains $\mathscr{F}$, the class of all finite soluble groups, and which is $\{S, D_0\}$-closed; that is, if $G \in \mathscr{X}$ and $H \leq G$ then $H \in \mathscr{X}$ and if $N_1, N_2 \in \mathscr{X}$ then $N_1 \times N_2 \in \mathscr{X}$.

We shall occasionally refer to specific subclasses and one should note the following possibilities for $\mathscr{X}$:

(1) $\mathscr{X} = \mathscr{S}_1$,
(2) $\mathscr{X} = \mathscr{P}$, the class of polycyclic groups,
(3) $\mathscr{X} = \mathscr{F}$, the class of finite soluble groups
(4) $\mathscr{X} = \mathscr{C}$, the class of Černikov (or extremal) soluble groups,
(5) $\mathscr{X} = \mathscr{M}$, the class of soluble minimax groups.

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