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SPECTRAL GEOMETRY OF THE SECOND VARIATION OPERATOR OF HARMONIC MAPS

BY

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Dedicated to Professor Shingo Murakami on the occasion of his sixtieth birthday

Introduction

In this paper, we deal with the inverse spectral problem of the Hessian (the so called Jacobi operator) of the energy of a harmonic map.

The spectral geometry of the Laplace-Beltrami operator has developed greatly during the last twenty years [3]. It is well-known [2], [20], [24] that if the spectrum Spec(Δ) of the Laplace-Beltrami operator Δ of a compact Riemannian manifold (M, g) coincides with the one of the standard sphere (S^n, can), $n < 7$, then (M, g) is isometric to (S^n, can). Since the Laplace-Beltrami operator of (M, g) can be regarded as the Jacobi operator of a constant map of (M, g) into a circle, it is reasonable to investigate the spectral geometry for the Jacobi operator of a harmonic map.

In fact, since the Jacobi operator J_ϕ of a harmonic map ϕ is a second order elliptic differential operator acting on the space of sections of the induced bundle of the tangent bundle of the target manifold, the spectrum Spec(J_ϕ) of J_ϕ becomes a discrete set of the eigenvalues with finite multiplicities. Directly applying Gilkey's results [11], [12] about the asymptotic expansion of the trace of the heat kernel of a certain differential operator of a vector bundle to our case, we can determine some geometric spectral invariants of the Jacobi operator (§§2, 3). Using these results, we obtain a series of geometric results distinguishing typical harmonic maps, i.e., (0) constant maps, (1) geodesics, (2) isometric minimal immersions, (3) holomorphic maps between Kaehler manifolds, and (4) Riemannian submersions all of whose fibers are minimal.

The analogue of spectral geometry for minimal submanifolds has been studied by H. Donnelly [7], and T. Hasegawa [15].

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