ALGEBRAIC ASPECTS OF CHEN'S TWISTING COCHAIN

BY

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In Memoriam Kuo-Tsai Chen

1. Introduction

The present volume is not only a memorial to K. T. Chen, but provides the opportunity, quoting [HT], for "a somewhat revisionist view of his approach to deRham homotopy theory". Since our own work has been heavily influenced by Chen's, we offer the following further but somewhat different insights into his work. In particular, one of Chen's major contributions was a method for computing the homology of the loop space on a manifold in terms of the homology of the manifold. He effected this via a differential on the tensor algebra T^a on the desuspension of the reduced homology of X. The differential satisfied a certain condition (detailed in §1.2 below) with respect to an element ω of the completed tensor product of the forms on X with $T^a s^{-1} H_+(X)$. Chen called ω a *formal power series connection*. By identifying this tensor product with an appropriate Hom, Chen's condition becomes that of a *twisting cochain*, as he implies in [C3] after Theorem 7.1. To provide a multiplicative chain equivalence $\Theta: C_*(\Omega X) \to (T^a s^{-1} H_+(X), \partial)$, Chen makes use of his iterated integrals.

Over the years, we have come to realize more fully the depth and significance of Chen's constructions. Initially, many of us focused excessively on two aspects of Chen's work: the analysis symbolized in his phrase "iterated integrals" and the homotopy theory of his alternative to Adams' cobar construction. From our current point of view, it is the algebraic aspects of Chen's work which have broadest significance and, within that context, some emphasis could be placed on his constructing a much smaller model than did Adams. The analysis used forced the homotopy theory into characteristic zero, whereas, with hindsight, we can see large portions of Chen's

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