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UNIQUENESS OF UNCONDITIONAL AND SYMMETRIC STRUCTURES IN FINITE DIMENSIONAL SPACES

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1. Introduction

The main target of the present paper is to study some questions concerning the uniqueness of symmetric and unconditional bases in the framework of the local theory of Banach spaces. Since the spaces under consideration are finite dimensional it is quite obvious that one cannot discuss problems of uniqueness for individual spaces but rather for families of such spaces. As we shall see in the sequel, the case of unconditional bases can be treated from different points of view.

The study of the uniqueness of symmetric bases for finite dimensional spaces was initiated in [8] and continued in [16] and [11] (see also [17]). Results concerning the uniqueness question in the setting of unconditional bases for finite dimensional Banach spaces were obtained in Schütt [16] and in [1].

In order to discuss our results as well as their connection with previously proved ones, we introduce the following definitions.

DEFINITION 1.1. (a) Let $\mathscr{F}$ be a family of finite dimensional Banach spaces each of which has a normalized 1-symmetric basis. We shall say that the members of $\mathscr{F}$ have a unique symmetric basis if there exists a function ψ: $[1,\infty) \to [1,\infty)$ such that, whenever $X \in \mathscr{F}$ has another normalized K-symmetric basis $(y_i)_{i=1}^n$, then $(y_i)_{i=1}^n$ is $\psi(K)$-equivalent to the given 1-symmetric basis.

(b) Let $\mathscr{F}$ be a family of finite dimensional Banach spaces each of which has a normalized 1-unconditional basis. We shall say that the members of $\mathscr{F}$ have an almost (somewhat) unique unconditional basis provided there exists a function φ: $[1,\infty) \times (0,1) \to [1,\infty)$ such that, whenever $X \in \mathscr{F}$ with the given 1-unconditional basis $(x_i)_{i=1}^n$ has also another normalized K-unconditional basis $(y_i)_{i=1}^n$ then, for any (some) $0 < \alpha < 1$, there exists a subset

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