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RIEMANNIAN FOLIATIONS ON SIMPLY CONNECTED MANIFOLDS AND ACTIONS OF TORI ON ORBIFOLDS

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1. Introduction

Basic properties of Riemannian foliations on simply connected manifolds have been established by P. Molino [Mol-1] and E. Ghys [Ghy]. In this paper we complete their results by showing a close relationship between such foliations and actions of tori on orbifolds.

As a general reference on Riemannian foliations, we refer to the book of P. Molino [Mol].

1.1. We first give a typical example where tori actions on orbifolds arise naturally.

Let H be a connected subgroup of the Lie group of isometries of an orientable Riemannian manifold Y. Let us assume that H acts locally freely on Y. Then the orbits under H of the points of Y are the leaves of a Riemannian foliation $\mathscr{F}$ on Y.

Assume that the closure $\overline{H}$ of H is compact. Let K be a maximal compact subgroup of H. As the Lie algebra of H is a compact Lie algebra ([Bki], Chap. IX), this maximal compact subgroup is unique, hence invariant in H. The quotient group $L = H/K$ is a dense abelian contractible subgroup of the compact group $\overline{L} = \overline{H}/K$ which must be isomorphic to a torus T^N of dimension N. The action of K on Y is also locally free; hence the orbits under K are the fibers of a generalized Seifert fibration on Y (i.e. a foliation whose leaves are compact with finite holonomy); its base space is naturally an oriented orbifold X whose underlying topological space is Y/K. The torus $T^N = \overline{H}/K$ acts effectively on X and the restriction of this action to the dense subgroup L is locally free. The orbits under L are the leaves of a foliation $\mathscr{F}_X$ on X, and the foliation $\mathscr{F}$ is the pull back of $\mathscr{F}_X$ by the projection p of Y on X.

Conversely, given an action of a torus T^N on an orientable orbifold X of dimension n (see 3.1 and 3.2) and a dense contractible subgroup L of T^N

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