ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 1, Spring 1991

CLASSIFICATION THEORY FOR A 1-ARY FUNCTION

BY

CARLO TOFFALORI

1. Introduction

Let T be a countable, complete 1st order theory; assume for simplicity that T has no finite models. It is sometimes possible to assign every model M of T an invariant—like a cardinal number, or something similar—such that $M \simeq M'$ if and only if M and M' have the same invariant. For instance, if T is the theory of algebraically closed fields of some fixed characteristic, then, for every $M \vDash T$, the isomorphism type of M is given by its transcendence degree. Let us say that T is classifiable if this assignment of invariants can be done. The classification problem, namely the problem of characterizing classifiable theories, was (almost) thoroughly solved by S. Shelah. It would be too long to report here the development and the results of Shelah's analysis (a clear introduction can be found in [B] and [Sh]); briefly summarizing, we can say that, it one agrees to the (reasonable) assumption

> T is classifiable if and only if there is an uncountable cardinal λ such that T has $< 2^{\lambda}$ non-isomorphic models of power λ,

then Shelah's main theorem states that

> T is classifiable if and only if T is superstable, presentable, shallow and satisfies the existence property.

The classification problem involves an obvious algebraic question, namely to find, given an elementary class K of 1st order structures, under which conditions a structure $M \in K$ satisfies "$Th(M)$ classifiable". This is the question we wish to deal with for the class K of all structures M with a 1-ary (total) function. Hence Sections 2 and 3 are devoted to translating in this context some of the basic notions of classification theory (like regular types, orthogonal types, and so on); §4 contains the main theorems on classifiable 1-ary functions, while, finally in Sections 5 and 6 we will study the characterization of non-multidimensional, unidimensional and categorical 1-ary functions.

Received April 6, 1988