ILLINOIS JOURNAL OF MATHEMATICS
Volume 35, Number 3, Fall 1991

ON UNCONDITIONALLY CONVERGING AND WEAKLY PRECOMPACT OPERATORS

BY

ELIAS SAAB AND PAULETTE SAAB[1]

Introduction

In [1], the authors showed that if F is a Banach space such that F^* has the Radon Nikodym property and contains no subspace isomorphic to l_1, and if G is any Banach space and Ω a compact Hausdorff space, then an operator $T\colon C(\Omega, F) \longrightarrow G$ is unconditionally converging if and only if its adjoint T^* is weakly precompact and they asked whether or not the result is still true if one assumes only that F^* does not contain a subspace isomorphic to l_1. In this paper we give a positive answer to their question. We actually prove a more general result, namely we show that if E, F and G are Banach spaces such that E^* is isometric to an L_1-space, and F^* contains no subspace isomorphic to l_1, a bounded linear operator $T\colon E \hat{\otimes}_\varepsilon F \longrightarrow G$ is unconditionally converging if and only if its adjoint T^* is weakly precompact. The methods used to prove this result allow us to extend the result of [17], namely we will show that if E^* is isometric to an L_1-space and F is any Banach space, then l_1 is isomorphic to a complemented subspace of $E \hat{\otimes}_\varepsilon F$ if and only if l_1 is isomorphic to a complemented subspace of F.

Notations and definitions

Let X and Y be two Banach spaces. A bounded linear operator $T\colon X \longrightarrow Y$ is said to be **unconditionally converging** if T sends weakly unconditionally Cauchy series $\sum_{n=1}^{\infty} x_n$ in X into unconditionally convergent series, and T is said to be **weakly precompact** if every bounded sequence $(x_n)_{n\geq 1}$ has a subsequence $(x_{n_k})_{k\geq 1}$ such that $(T(x_{n_k}))_{k\geq 1}$ is weakly Cauchy. It follows from Rosenthal l_1 Theorem (see [16] or [9]) that T is weakly precompact if and only if the image by T of the unit ball of X does not contain a sequence equivalent to the l_1 basis. It follows from [8] see also

---

Received December 21, 1989.

1980 Mathematics Subject Classification (1985 Revision). Primary 46E40, 46G10; Secondary 28B05, 28B20.

[1]Supported in part by a grant from the National Science Foundation.