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QUATERNION L-VALUE CONGRUENCES AND GOVERNING FIELDS OF S-CLASS GROUPS

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0. Introduction

The goal of Galois Module Theory is to describe the algebraic structure of modules acted on by Galois group rings. A fundamental result was M.J. Taylor's proof (cf. [F3] for a full discussion) that the ring of integers in a tamely ramified extension of number fields is a free Galois module if and only if a certain *analytic* invariant, constructed from root numbers of Artin L-functions, is trivial. Noticing similarities between the above setting and that of J. Tate's approach to the Stark conjectures in [T], T. Chinburg in [Ch1] conjectured a similar relationship for the Galois module structure of certain S-units. His proof in [Ch2] of this conjecture for a certain family of quaternion extensions (which are the first technically interesting case) relied upon establishing the existence of a governing field for the variation of the structure of the S-class group when S is the set of ramified primes of these extensions. By different techniques, Chinburg in [Ch3] was able to find L-value congruences for a subset of the fields considered in [Ch2] which by our results lead to a precise governing field.

We give a precise governing field for the variation of the Galois module structure of the S-class group for all of the quaternion extensions considered in [Ch2]. Using G. Gras's analytic genus theory, we proceed to give a precise governing field in the context of a previously unstudied family of quaternion extensions. This new result suggests that one now study the algebraic structure for these extensions.

Our approach uses congruence techniques to replace longer classfield theoretic arguments showing that a particular set of primes generates the 2-Sylow subgroup of the ideal classgroup of an extension. That stronger congruences then determine the existence of the governing fields follows from an observation of [Ch2], as discussed in our Section 6.

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