CYCLIC VECTORS FOR INVARIANT SUBSPACES IN SOME CLASSES OF ANALYTIC FUNCTIONS

BY

A. MATHESON

1. Let ψ be a positive increasing function on $(0, \infty)$ such that

$$\lim_{t \downarrow 0} \psi(t) = 0, \, \psi(t) = \psi(1) \text{ for } t > 1 \quad \text{and} \quad \int_0^1 \frac{1}{\psi(t)} \, dt < \infty.$$

Define

$$M_{\psi} = \left\{ f \in H^{\infty} | M_{\infty}(f', r) = o\left(\frac{\psi(1-r)}{1-r}\right) \right\}$$

and

$$L_{\psi} = \left\{ f \in H^{\infty} \middle| \int_0^1 \frac{M_{\infty}(f', r)}{\psi(1 - r)} \, dr < \infty \right\},$$

where H^{∞} is the space of bounded analytic functions on the unit disk, and

$$M_{\infty}(g,r) = \sup_{|z|=r} |g(z)| = \sup_{|z|\leq r} |g(z)|.$$

Each of the spaces M_{ψ} and L_{ψ} becomes a Banach algebra under the norms

$$\begin{split} \|f\|_{M_{\psi}} &= \|f\|_{\infty} + \sup_{0 < r < 1} \frac{(1 - r)M_{\infty}(f', r)}{\psi(1 - r)}, \\ \|f\|_{L_{\psi}} &= \|f\|_{\infty} + \int_{0}^{1} \frac{M_{\infty}(f', r)}{\psi(1 - r)} \, dr \end{split}$$

respectively. Since $M_{\infty}(f', r)$ is increasing and $\psi(1 - r)$ is decreasing, it is

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