ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 1, Spring 1992

A LOW INTENSITY MAXIMUM PRINCIPLE FOR BI-BROWNIAN MOTION

BY

THOMAS S. SALISBURY[1]

1. Introduction

Bi-Brownian motion is the process $Z(s,t) = (X(s), Y(t))$, where X and Y are independent d-dimensional Brownian motions started from $\mathbf{0}$. Here $d \geq 3$. Studying a pair of Brownian motions as a two-parameter process has proved useful both in probability (for example path intersections—see [DS] or [FS]) and analysis (see [GS] or [W1]).

Estimates of the hitting probabilities for Z were given in [FS]. They use capacity (defined analytically) for the kernel $u = v \otimes v$, where $v(x,y) = c(d)|x-y|^{2-d}$ is the Green function for Brownian motion. In other words, if $z = (x,y)$ and $z' = (x', y')$ then

$$u(z, z') = v(x, x')v(y, y').$$

While the capacity theory for u behaves well (see also [F] and [O]), the same cannot be said for other potential theoretic objects involving u. The principal cause is that the maximum principle fails badly (see §4). Since the maximum principle is closely tied to the strong Markov property this failure is not unexpected. It is however possible to retain it in a weakened form.

We will establish a "low-intensity" version of the bounded maximum principle for Z, to the effect that if $U\mu = \int u(\cdot, z)\mu(dz)$ is bounded on K and $\mathbf{0}$ is far from K, in the sense that the probability of hitting K starting from $\mathbf{0}$ is small, then $U\mu(\mathbf{0})$ can't be large. This would not be hard if "far" were interpreted using the Euclidean distance. In contrast, our condition can be thought of as allowing K to be thin at $\mathbf{0}$ (in a fairly stringent sense). This permits consideration of sets like thorns or fractal dusts, and forces us to

Received September 15, 1989.

1980 Mathematics Subject Classification (1985 Revision). Primary 60J65; Secondary 60G60, 31B30.

[1]Research supported in part by a grant from NSERC.