ILLINOIS JOURNAL OF MATHEMATICS
Volume 36, Number 2, Summer 1992

COPIES OF l_∞ IN KÖTHE SPACES OF VECTOR VALUED FUNCTIONS

BY

GIOVANNI EMMANUELE[1]

Let (S, Σ, μ) be a σ-finite complete measure space and X be a Banach space. Recently the following result has appeared

THEOREM 1 [7]. *Let $1 \leq p < \infty$. Then l_∞ embeds into $L^p(\mu, X)$ if and only if it embeds into X.*

The purpose of this note is to extend Theorem 1 to a more general class of vector-valued functions; namely, Köthe spaces $E(X)$ of vector-valued functions. Specifically, we show that l_∞ embeds into $E(X)$ if and only if it embeds into either E or X. We recall that $L^p(\mu, X)$ spaces as well as Orlicz or Musielak-Orlicz spaces of vector-valued functions are special cases of Köthe spaces.

Before giving our result, we need some definitions and results. Let $\mathscr{M}(S) = \mathscr{M}$ be the space of Σ-measurable real valued functions with functions equal μ-almost everywhere identified. A Köthe space E [6] is a Banach subspace of $\mathscr{M}$ consisting of locally integrable functions such that (i) if $|u| \leq |v|$ μ. a.e., with $u \in \mathscr{M}$, $v \in E$ then $u \in E$ and $\|u\|_E \leq \|v\|_E$, (ii) for each $A \in \Sigma$, $\mu(A) < \infty$, the characteristic function χ_A of A is in E. Köthe spaces are Banach lattices if we put $u \geq 0$ when $u(s) \geq 0$ μ. a.e. Furthermore, Köthe spaces are σ-complete Banach lattices. The following theorems will be utilized in the sequel.

THEOREM 2 [5]. *Given a Köthe space E, there exists an increasing sequence (S_n) in Σ with $\mu(S_n) < \infty$ and $\mu(S \setminus \bigcup_{n \in N} S_n) = 0$ for which the following chain of continuous inclusions holds:*

$$L^\infty(S_n) \subset E(S_n) \subset L^1(S_n).$$

Received August 17, 1990.

1980 Mathematics Subject Classification (1985 Revision). Primary 46E30; Secondary 46E40, 46B25.

[1]Work performed under the auspices of G.N.A.F.A. of C.N.R. and partially supported by M.U.R.S.T. of Italy.