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ENDOMORPHISMS OF CERTAIN IRRATIONAL ROTATION C*-ALGEBRAS

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1. Preliminaries for quadratic irrational numbers

First we will give definitions and well known facts on quadratic irrational numbers. Let $GL(2, \mathbf{Z})$ be the group of all 2×2-matrices over $\mathbf{Z}$ with determinant ± 1. Let

$$g = \begin{bmatrix} k & l \\ m & n \end{bmatrix} \in GL(2, \mathbf{Z})$$

and θ be an irrational number. We define

$$g\theta = \frac{m + n\theta}{k + l\theta}$$

and we call g a *fractional transformation*. Furthermore if

$$g \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

then we say that g is *non-trivial*.

Let $\mathbf{Q}$ be the ring of rational numbers. We suppose that θ is a quadratic irrational number. If $\theta = x + y\sqrt{d}$ where $x, y \in \mathbf{Q}$ and $d \in \mathbf{N}$, then we define $\theta' = x - y\sqrt{d}$ and we call θ' the *conjugate* of θ. We say that θ is reduced if $\theta > 1$ and $-1 < \theta' < 0$ where θ' is the conjugate of θ.

For any quadratic irrational number θ there are a fractional transformation

$$g = \begin{bmatrix} k & l \\ m & n \end{bmatrix} \in GL(2, \mathbf{Z})$$

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