ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, Number 1, Spring 1993

LOCAL BOUNDARY REGULARITY OF THE BERGMAN PROJECTION IN NON-PSEUDOCONVEX DOMAINS

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1. Introduction

Let Ω be a bounded domain in $\mathbf{C}^n$ with smooth boundary. The Bergman projection P associated to Ω is the orthogonal projection from the space of square-integrable functions on Ω onto the subspace consisting of holomorphic functions. The global or local boundary regularity of the Bergman projection, as well as the boundary extendibility of the Bergman kernel function $K(z,w)$, was proved to have important applications in studying the boundary behavior of biholomorphic and proper holomorphic mappings of Ω [5], [8], [9], [15]. If Ω is a pseudoconvex domain, many results have been obtained as consequences of the $\bar{\partial}$-Neumann theory. For instance, the Bergman projection for a pseudoconvex domain is locally regular, or, satisfies certain pseudolocal estimates at all boundary points of finite type in the sense of D'Angelo [14]. Also the main theorem in [19] states that $K(z,w)$ is smooth in both variables up to the boundary off the boundary diagonal in a strictly pseudoconvex domain. In [3] or [10] the same conclusion has been generalized for weakly pseudoconvex domains of finite type. It has also been shown for smoothly bounded Reinhardt domains [7], which are not necessarily pseudoconvex, that the Bergman projection is globally regular and that the Bergman kernel function behaves nicely on the boundary. Namely, the well-known condition R is satisfied (see Definition 2.1). Thus any derivative of $K(z,w)$ in the z-variable has uniform polynomial growth in the w-variable. See [1] and [4] for some other types of domains that satisfy condition R.

When the smoothly bounded domain is assumed to be arbitrary, little is known about the boundary regularity of the Bergman projection. In [2], Barrett presented a non-pseudoconvex bounded Hartogs domain D with smooth boundary which does not satisfy condition R. Actually, in his example the subspace of bounded holomorphic functions is not dense in the space $H(D)$ of square-integrable holomorphic functions. So there is a smooth function ϕ which is compactly supported in D such that the Bergman projection $P\phi$ of ϕ is not bounded. It is easily seen that for some point w in

Received April 12, 1991.
1991 Mathematics Subject Classification. Primary 32H10.