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A CHARACTERIZATION OF THE LEADING COEFFICIENT OF NEVANLINNA'S PARAMETRIZATION

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1. Introduction

Let H^∞ be the Banach space of all bounded analytic functions in the open unit disc D, with the norm $\|f\|_\infty = \sup\{|f(z)|: z \in D\}$. Given two sequences of points $\{z_n\}, \{w_n\}$ in D, the classical Pick-Nevanlinna problem consists on finding analytic functions $f \in H^\infty$ satisfying $\|f\|_\infty \leq 1$ and $f(z_n) = w_n$, $n = 1, 2, \dots$. We will denote it as follows:

$$(*) \quad \text{Find } f \in H^\infty,\ \|f\|_\infty \leq 1,\ f(z_n) = w_n, \qquad n = 1, 2, \dots .$$

Pick and Nevanlinna found necessary and sufficient conditions in order that such an analytic function exists. Let E be the set of all solutions of the problem $(*)$. Nevanlinna showed that if E has more than one element, there exist analytic functions p, q, r, s in D such that

$$E = \left\{ f \in H^\infty : f = \frac{p\varphi + q}{r\varphi + s},\ \varphi \in H^\infty,\ \|\varphi\|_\infty \leq 1 \right\} \tag{1.1}$$

$$ps - qr = B \tag{1.2}$$

where B is the Blaschke product with zeros $\{z_n\}$. See [2, p. 165] for the proof. Let us remark that there is no explicit formula for the coefficients p, q, r, s in terms of the sequences $\{z_n\}, \{w_n\}$.

We will say that a Pick-Nevanlinna problem $(*)$ with more than one solution has the function s as leading coefficient if s is analytic in D and there exist analytic functions p, q, r in D such that if E is the set of all solutions of $(*)$, the functions p, q, r, s verify (1.1) and (1.2).

In this note, fixed a Blaschke sequence $\{z_n\}$ in D, we get a characterization of the functions that can appear as leading coefficients of Pick-Nevanlinna problems $(*)$.

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