ILLINOIS JOURNAL OF MATHEMATICS
Volume 37, Number 2, Summer 1993

LUSTERNIK-SCHNIRELMANN COCATEGORY

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1. Introduction

In the last ten years there has been a revival of interest in the (Lusternik-Schnirelmann) category of a topological space X. Recall the classical definition: a space X has category $\leq n$ if and only if it can be covered by $n + 1$ open sets, each of which is contractible in X. This revival came from rational homotopy theory. Finite category seems to be the right finiteness restriction on a rational space.

Cocategory is much less well understood than category. In fact it is not clear that we have the right definition of it yet. The first attempt to define cocategory was made by Ganea in 1960 [4] and [5]. His invariant, which we will call inductive cocategory, satisfied many of the properties for which one would hope. The most fundamental of these are the following:

1. The spaces of cocategory one are the H-spaces.
2. In a space of cocategory n, Whitehead products in the homotopy of length greater than n vanish.
3. In a fibration, the cocategory of the fiber can be no bigger than the cocategory of the total space plus one.

However, inductive cocategory has a rather inscrutable definition. Not many papers were written about it. Then in his Oxford thesis, Hopkins pointed out that there is more than one natural choice for a definition of cocategory. He introduced symmetric cocategory, which he proves satisfies the first two properties above. He also shows that symmetric cocategory is at least as big as inductive cocategory, but he is unable to determine if they are equal (see [8]).

At about the same time Sbaï investigated rational cocategory. There is an obvious choice for a rational definition of cocategory, using the Quillen model and dualizing a definition of Félix and Halperin in [2]. Sbaï was trying

Received June 14, 1991.
1991 Mathematics Subject Classification. Primary 55M30.