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THE HILBERT TRANSFORM ALONG CURVES THAT ARE ANALYTIC AT INFINITY

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1. Introduction

It is known that if B denotes the unit ball of $\mathbf{R}^m, \gamma: B \to \mathbf{R}^n$ is an analytic function, $\gamma(0) = 0$, and k is a $C^\infty(\mathbf{R}^m - \{0\})$ function, homogeneous of degree $-m$, then the operator given by $Tf(x) = p \cdot v \cdot \int_B f(x - \gamma(t))k(t)\,dt$ is bounded on $L^p(\mathbf{R}^n)$, $1 < p < \infty$. See for example [2], [9]. We observe that in this case γ is "approximately homogeneous" at the origin in the sense given in [10].

The purpose now is to consider the analogous problem at infinity, for the case $m = 1$. More precisely we prove the following:

THEOREM 1.1. *Let $B^C = \{t \in \mathbf{R}: |t| > 1\}$ and let $\gamma: B^C \to \mathbf{R}^n$ be defined by*

$$\gamma(t) = \left(t^{a_1} + \alpha_1(t), \dots, t^{a_n} + \alpha_n(t)\right), a_i \in N, \qquad a_1 < \cdots < a_n,$$

where α_i is a real analytic function on B^C, $\alpha_i(t) = h_i(t) + P_i(t)$ with h_i analytic at infinity, and P_i a polynomial of degree at most $a_i - 1$. Then the operator

$$\mathscr{H}_\gamma f(x) = p \cdot v \cdot \int_{B^C} f(x - \gamma(t))\frac{dt}{t}$$

is bounded on $L^p(\mathbf{R}^n)$, $1 < p < \infty$.

This result still holds if $\gamma(t) = (\gamma_1(t) + \alpha_1(t), \dots, \gamma_n(t) + \alpha_n(t))$ where $\gamma_i(t)$ are homogeneous functions of degree a_i, $a_i \in \mathbf{R}$, $1 \le a_1 < \cdots < a_n$, and asking weaker conditions about the behavior at infinity of $\alpha_i(t)$.

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