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ON THE SET OF TOPOLOGICALLY INVARIANT MEANS ON THE VON NEUMANN ALGEBRA $VN(G)$

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1. Introduction

The study of the cardinality of the set of invariant means on a group was initiated by Day [3] and Granirer [8]. In 1976, Chou [1] showed that for a discrete infinite amenable group G the cardinality of the set $ML(G)$ of all left invariant means on $l^\infty(G)$ is $2^{2^{|G|}}$. Later, Lau and Paterson [20] proved that if G is a noncompact amenable locally compact group, then the set $MTL(G)$ of all topologically left invariant means on $L^\infty(G)$ has cardinality $2^{2^{d(G)}}$, where $d(G)$ is the smallest cardinality of a covering of G by compact sets. (Of course, when G is compact, $MTL(G)$ is the singleton containing only the normalized Haar measure of G). For results on the size of the set $ML(G) \setminus MTL(G)$, see Granirer [9], Rudin [29], and Rosenblatt [26]. See also Yang [32] and Miao [21] for some recent developments in certain related aspects. We refer the readers to the books of Pier [23] and Paterson [22] for more details on the study of the size and the structure of the set of invariant means on groups and semigroups.

Let G be a locally compact group, $A(G)$ the Fourier algebra of G, $VN(G)$ the von Neumann algebra defined by the left regular representation $\{\rho, L^2(G)\}$ and $TIM(\hat{G})$ the set of all topologically invariance means on $VN(G)$. The set $TIM(\hat{G})$ was first studied by Dunkl and Ramirez for compact groups. They showed [4] that if G is an infinite compact group, then $|TIM(\hat{G})| \geq 2$. Renaud [25, Theorem 1] proved that there exists a unique topologically invariant mean on $VN(G)$ when G is discrete. In Theorem 1 of [10], Granirer showed the following: if G is non-discrete and second countable (i.e., there is a countable basis for open sets in G), then $TIM(\hat{G})$ is not norm separable. A stronger results was obtained by Chou in [2, Theorem 3.3]: if G is non-discrete and metrizable, then there exists a linear isometry of $(l^\infty)^*$ into $VN(G)^*$ which embeds a "big subset" (having cardinality 2^c) of $(l^\infty)^*$ into $TIM(\hat{G})$. See also Granirer [13, p.172–173] for the discussion on the set $TIM_p(\hat{G})$ of topologically invariant means on $A_p(G)^*$, where $A_p(G)$ is

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