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FACTORIZATION OF COMPOSITION OPERATORS THROUGH BLOCH TYPE SPACES

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Introduction

One way to compare function theoretic properties of analytic functions and functional analytic properties of linear operators is through "change of variable" type formulae. The corresponding operators are known as *composition operators*; they have been studied on various classical function spaces, in particular Hilbert function spaces. In this paper we continue investigating composition operators within the framework of general Hardy spaces on the open unit disk.

We are going to identify those composition operators C_φ, say from H^1 to H^1, which allow a canonical factorization $X_1 \to H^\beta$ for some $\beta > 0$ where X_1 is isometrically isomorphic to the classical Bloch space B. More precisely, our result characterizes φ's such that $f \mapsto f' \circ \varphi$ defines a bounded linear map $B \to H^\beta$, and we shall obtain analogous statements for H^p-spaces when $p > 1$ by replacing B with appropriate analytic Lipschitz spaces. As it will turn out, such operators are not only bounded but enjoy nuclearity properties similar to those of diagonal operators $l^\infty \to l^p$.

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Preliminaries

We shall mainly work with classical Hardy spaces

$$H^p \ (0 < p \leq \infty).$$

Recall that H^p consists of all analytic functions f on the open unit disk

$$D := \{z \in \mathbf{C}: |z| < 1\}$$

in the complex plane which satisfy

$$\|f\|_p := \sup_{r<1} \left(\int_{-\pi}^{\pi} |f(re^{it})|^p \frac{dt}{2\pi} \right)^{1/p} < \infty$$

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