ILLINOIS JOURNAL OF MATHEMATICS
Volume 39, Number 3, Fall 1995

STABILITY OF SOME TYPES OF RADON-NIKODYM PROPERTIES

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I. Introduction

Let X be a Banach space, let (Ω, Σ, μ) be a measure space and let $1 \leq p \leq \infty$. $L^p(\mu, X)$ will denote the Banach space of all (classes of) X-valued μ-p-Bochner integrable with the usual norm. If X is scalar field, then we will write $L^p(\mu)$ for $L^p(\mu, X)$.

In this note, we consider some types of Radon Nikodym properties associated with subsets of countable discrete abelian group (type I-Λ-RNP and type II-Λ-RNP) which generalize the usual Radon Nikodym property and the Analytic Radon Nikodym property. These properties were introduced by Dowling [D1] and Edgar [E].

In [D1], it is shown that if Λ is a Riesz set then $L^1[0,1]$ has type I-Λ-RNP. It is then natural to ask if these two properties pass from X to $L^p(\mu, X)$. Dowling proved in [D1] that if Λ is a Riesz subset of $\mathbf{Z}$, then $L^1(\mathbf{T}, X)$ has type II-Λ-RNP whenever X does. In this paper, we will show that the same result holds regardless of the group G and the measure space (Ω, Σ, μ). We will give also some generalization for non Riesz sets.

We will discuss also when type I-Λ-RNP and type II-Λ-RNP pass from a Banach space X to $C_\Lambda(G, X)$ if Λ is a Rosenthal set. Other related results are obtained.

All unexplained terminologies can be found in [D] and [DU].

II. Preliminaries and definitions

Throughout this paper G will denote a compact metrizable abelian group, $\mathscr{B}(G)$ is the σ-algebra of the Borel subsets of G, and λ the normalized Haar measure on G. We will denote by Γ the dual group of G, i.e., the set of continuous homomorphisms $\gamma: G \to \mathbf{C}$ (Γ is a countable discrete abelian group).

Let X be a Banach space and $1 \leq p \leq \infty$, we will denote by $L^p(G, X)$ the usual Bochner function spaces for the measure space $(G, \mathscr{B}(G), \lambda)$, $M(G, X)$ the space of X-valued countably additive measure of bounded variation,

Received April 5, 1993.

[1]This work will constitute a portion of the Ph.D. thesis of the first named author at the University of Missouri-Columbia.

1991 Mathematics Subject Classification. Primary 46E40, 46G10; Secondary 28B05, 28B20.