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FREE GROUPS AND UNIFICATION IN $\mathfrak{A}_m\mathfrak{A}_2$

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1. Introduction

In [3], the order of the free r-generated group in the variety generated by a dihedral group D of order $2^{d+1}e$ (where e is odd) is determined to be $2^{r+s}e^{r'}$ where

$$r' = 2^r(r-1) + 1$$
$$s = \sum_{t=2}^{d} (d+1-t)(t-1)\binom{r+1}{t}$$

(there is a typographical error in the definition of r' in [3]).

The proof of this result depends on a structure theorem for the variety generated by D;

$$\operatorname{var} D = \begin{cases} \mathfrak{A}_e\mathfrak{A}_2 & \text{when } d < 2, \\ \mathfrak{A}_e\mathfrak{A}_2 \vee (\mathfrak{A}_{2^{d-1}}\mathfrak{A}_2 \wedge \mathfrak{N}_d) & \text{when } d \geq 2. \end{cases}$$

Here the notation is as in [7]; in particular $\mathfrak{A}_n$ is the variety of abelian groups of exponent dividing n, $\mathfrak{N}_c$ is the variety of nilpotent groups of class c, and if $\mathfrak{A}$ and $\mathfrak{B}$ are varieties, then $\mathfrak{A}\mathfrak{B}$ is the variety of all groups which are an extension of a group in $\mathfrak{A}$ by one in $\mathfrak{B}$.

In the case $d \geq 2$ the calculation of the order then depends on the results in [4] which give a normal form description for elements of the free groups in the varieties $\mathfrak{A}_{p^\alpha}\mathfrak{A}_p$ (where p is a prime).

In this paper we will restrict our attention to the first case, $d < 2$. As a matter of personal preference we use m rather than e for the odd part, and so our goals are to describe the free groups of the variety

$$\mathfrak{A}_m\mathfrak{A}_2$$

where m is odd, and to determine the *unification type* of this variety, which in this context amounts to describing a single most general solution to any system of equations

$$\Sigma = \{t_1(x) = 1, t_2(x) = 1, \ldots, t_n(x) = 1\}$$

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