ILLINOIS JOURNAL OF MATHEMATICS
Volume 39, Number 3, Fall 1995

DE BRANGES SPACES CONTAINED IN SOME BANACH SPACES OF ANALYTIC FUNCTIONS

DINESH SINGH AND SANJEEV AGRAWAL

1. Introduction

L. de Branges has proved in Theorem 15 of [2] an invariant subspace theorem which generalizes not only Beurling's famous theorem [1] but also its generalizations due to Lax [7] and Halmos [4]. The scalar version of the theorem says:

THEOREM A. *Let M be a Hilbert space contractively contained in the Hardy space H^2 of the unit D such that $S(M) \subset M$ (where S is the operator of multiplication by the coordinate function z) and S acts as an isometry on M. Then there exists a unique b in the unit ball of H^∞ such that*

$$M = b(z)H^2.$$

Further,

$$\|bf\|_M = \|f\|_{H^2}.$$

In this note we characterize those Hilbert spaces M which are algebraically contained in various Banach spaces of analytic functions on the unit disc D. We drop the contractivity requirement on M (no continuity assumptions are made on the inclusion relation). Thus even in the particular case of $M \subset H^2$, we obtain an extension of de Branges Theorem by having characterized the class of all Hilbert spaces which are vector subspaces of H^2 and on which S acts as an isometry. See Corollaries 5.1 and 4.1.

2. Preliminary notations, definitions and results

Let D be the unit disc in the complex plane and H^p $(0 < p \leq \infty)$ the well known Hardy spaces on D. Let L^p $(0 < p \leq \infty)$ be the familiar Lebesgue spaces on the unit circle T. It is well known that H^p can be viewed as a space of functions on T for each p. The Dirichlet space A^2 consists of all analytic functions $f(z)$ such that

$$\int_D |f'(z)|^2 \, dx\, dy < \infty.$$

Received January 5, 1993.
1991 Mathematics Subject Classification. Primary 47B37; Secondary 46E15.