ILLINOIS JOURNAL OF MATHEMATICS
Volume 40, Number 2, Summer 1996

ENERGY MINIMIZING SECTIONS OF A FIBER BUNDLE

SHIAH-SEN WANG

0. Introduction

Interior partial regularity for minimizers of functionals having nonquadratic growth between Riemannian manifolds has been extensively studied. See [2], [6], [8], [9] and references therein for details. Here we study sections of a fiber bundle X that locally minimize the L^p norm of the gradient among all $L^{1,p}_{\mathrm{loc}}$ sections when $p \in (1, \infty)$.We show that such a local minimizing section is Hölder continuous everywhere except a closed subset Z of the base manifold M, and that the set Z has Hausdorff dimension at most $m - [p] - 1$, where m is the dimension of M.

It is a well-known topological fact that there is no continuous unit tangent vector field on an even-dimensional sphere; thus continuity of a local minimizing section on *all* of M may be impossible by topological obstructions. In the trivial bundle case, i.e., $X = M \times N$ with N as the fiber, and $p = 2$, the problem studied here can be easily reduced to study minimizing harmonic maps from M to N; therefore, continuity of local minimizing sections may be impeded by energy considerations (see [7]), even without the topological obstructions.

In contrast with harmonic sections (see [1], 2.39), we *do* include the "horizontal" energy in the energy functional. This causes a major problem in proving the partial regularity for minimizing sections of the simplest form of functionals having nonquadratic growth discussed here because we have to deal with the map constraint—the projection map π of the fiber bundle.

The methods used to prove the results are described as follows:

In Section 1, first we locally associate an $L^{1,p}$ section $\widetilde{v}$ with each map $v \in L^{1,p}(\Omega, N)$ for some bounded open subset Ω of M by the local trivialization property of the bundle. Then we construct a new functional $\mathcal{G}$ defined on $L^{1,p}(\Omega, N)$ from the original one—the L^p norm of the gradient. Via this reformulation, we can study $\mathcal{G}$-minimizers with submanifold N constraint instead of p-energy minimizers with the mapping constraint π.

In Section 2, we prove that small normalized p-Dirichlet energy of a $\mathcal{G}$-minimizer u implies Hölder continuity using the De Giorgi blowing up argument outlined in Luckhaus' paper [9]—where he studies general functionals with nice blow-ups. The key ingredients of the proof are Lemma 2 and Lemma 3. We show the blow-up functional $\mathcal{F}$ of $\mathcal{G}$ is nice (in fact, our blow-up functional $\mathcal{F}$ is nicer than the one

Received September 8, 1994.
1991 Mathematics Subject Classification. Primary 58E20; Secondary 49K20