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RIEMANNIAN SUBMERSIONS WHICH PRESERVE THE EIGENFORMS OF THE LAPLACIAN

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Let $\pi: Z \to Y$ be a Riemannian submersion where Y and Z are closed Riemannian manifolds. Let $E(\lambda, \Delta_p^Y) \subset C^\infty \Lambda^p Y$ and $E(\lambda, \Delta_p^Z) \subset C^\infty \Lambda^p Z$ be the eigenspaces of the p form valued Laplacians on Y and on Z. We say the pullback

$$\pi^*: C^\infty \Lambda^p Y \to C^\infty \Lambda^p Z \tag{1}$$

preserves the p eigenforms of the Laplacian if for any $\lambda \in \mathbb{R}$, there exists $\mu(\lambda) \in \mathbb{R}$ so that

$$\pi^* E(\lambda, \Delta_p^Y) \subseteq E(\mu(\lambda), \Delta_p^Z); \tag{2}$$

in other words $\pi^*\Phi$ is an eigenform of Δ_p^Z, although with a possibly different eigenvalue, for every eigenform Φ of Δ_p^Y.

THEOREM 1. *The following conditions are equivalent*:

(a) *The fibers of π are minimal submanifolds.*
(b) $\Delta_0^Z \pi^* = \pi^* \Delta_0^Y$.
(c) π^* *preserves the eigenfunctions of the Laplacian* Δ_0^Y.

THEOREM 2. *The following conditions are equivalent*:

(a) *The fibers of π are minimal submanifolds and the horizontal distribution of π is integrable.*
(b) *For all* $0 \le p \le \dim(Y)$, $\Delta_p^Z \pi^* = \pi^* \Delta_p^Y$.
(c) *There exists p with $1 \le p \le \dim(Y)$ such that π^* preserves the p eigenforms of the Laplacian* Δ_p^Y.

These results deal with the totality of the eigenspaces; the following result deals with a single eigenform.

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