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APPROXIMATION BY ENTIRE FUNCTIONS AND ARAKELYAN-TYPE EXAMPLES FOR MOVING TARGETS

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Introduction

In this note we apply the approximation techniques developed in [17], together with explicit formulas for the solution of $\bar{\partial}$-equations, to construct examples in value distribution theory of one and several complex variables. From our point of view the suggested method has three main advantages: it is very natural, it is simple and explicit and it is powerful enough to derive new results. The general approximation scheme is described in Section 2.

Our aim is to construct examples of entire functions with given deficient values. It is well-known [16], [5] (see also [4], [6] for the case of meromorphic functions) that if the sum of deficiencies of an entire function is maximal, i.e.,

$$\sum_{a \in \mathbf{C}} \delta_f(a) = 1,$$

then f has integral order ρ and at most ρ deficient values $a_1, \ldots, a_\rho$, each having deficiency $1/\rho$.

Suppose that an integer ρ and the values $a_1, \ldots, a_\rho$ are given and one would like to have an example of an entire function of order ρ with these deficient values and maximal sum of deficiencies. Examples of such type have been constructed first by R. Nevanlinna [14]. The above mentioned approximation scheme permits an explicit formula to construct such examples. Moreover, the scheme is valid not only for constant deficient values, but also for small deficient functions (the so called slowly moving targets); that is, instead of just a-points, one studies the solutions of

$$f(z) = a(z),$$

for such $a(z)$ that

$$T_a(r) = o(T_f(r)), \quad r \to \infty.$$

In [13] (see [9] for the case of meromorphic functions) it was proved that the above properties of an entire function with maximal sum of deficient values take place also in the case of small functions. However, to the best of our knowledge,

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