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THE NEAR RADON-NIKODYM PROPERTY IN LEBESGUE-BOCHNER FUNCTION SPACES

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1. Introduction

Let X be a Banach space, $(\Omega, \Sigma, \lambda)$ be a finite measure space and $1 \leq p < \infty$. We denote by $L^p(\lambda, X)$ the Banach space of all (classes of) λ-measurable functions from Ω to X which are p-Bochner integrable with its usual norm $\|f\|_p = (\int \|f(\omega)\|^p \, d\lambda(\omega))^{1/p}$. If X is the scalar field then $L^p(\lambda, X)$ will be denoted by $L^p(\lambda)$.

The relationship between Radon-Nikodym type properties for Banach spaces and operators with domain $L^1[0, 1]$ is classical in theory of vector-measures. Such connections have been investigated by several authors. In [17], Kaufman, Petrakis, Riddle and Uhl introduced and studied the notion of nearly representable operators (see definition below). They isolated the class of Banach spaces X for which every nearly representable operator with range X is representable. Such Banach spaces are said to have the Near Radon-Nikodym Property (NRNP). It was shown in [17] that every Banach lattice that does not contain any copy of c_0 has the NRNP; in particular L^1-spaces have the NRNP. A question that arises naturally from this fact is whether the Lebesgue-Bochner space $L^1(\lambda, X)$ has the NRNP whenever X does. Let us recall that the answers to similar questions about related properties such as the Radon-Nikodym property (RNP), the Analytic Radon-Nikodym property (ARNP) and the complete continuity property (CCP) are known for Bochner spaces (see [24], [9] and [20] respectively). We also remark that Hensgen [14] observed that (as in the scalar case) $L^1(\lambda, X)$ has the NRNP if X has the RNP.

In this paper, we show that the Near Radon-Nikodym property can indeed be lifted from a Banach space X to the space $L^1(\lambda, X)$. Our proof relies on a representation of operators from L^1 into $L^1(\lambda, X)$ due to Kalton [16] and properties of operator-valued measurable functions along with some well known characterization of integral and nuclear operators from L^∞ into a given Banach space.

Our notation is standard Banach space terminology as may be found in the books [6], [7] and [26].

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