AN ISOMORPHISM THEOREM FOR CERTAIN FINITE GROUPS[1]

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Introduction

Let K be a finite field of characteristic p, and let $SL(2, K)$ be the unimodular group of 2 by 2 matrices of determinant one with coefficients in K. We shall be concerned with a finite group G which satisfies a list of axioms which say, roughly speaking, that G is generated by a certain number of subgroups which are homomorphic images of $SL(2, K)$, and that G has p-Sylow subgroups X and Y with certain special properties. We prove that all the finite simple groups G' defined by Chevalley [2] with respect to a finite field K of characteristic $p \geqq 5$, and the variations of them defined by Steinberg [13], satisfy our axioms.

The first main result concerns two finite groups G and $\bar{G}$ satisfying the axioms, and generated by subgroups $\phi_1(SL(2, K_1)), \cdots, \phi_l(SL(2, K_l))$ and $\bar{\phi}_1(SL(2, K_1)), \cdots, \bar{\phi}_l(SL(2, K_l))$, respectively, where the K_i are subfields of K, and the ϕ_i and $\bar{\phi}_i$ are homomorphisms of $SL(2, K_i)$ into G and $\bar{G}$. Let M and $\bar{M}$ be irreducible right ΩG- and $\Omega\bar{G}$-modules respectively, where Ω is an arbitrary extension field of K, and ΩG, $\Omega\bar{G}$ denote the group algebras over Ω of G and $\bar{G}$. A sufficient condition is obtained in order that there exist an Ω-isomorphism $S : M \to \bar{M}$ such that

$$m\phi_i(g)S = (mS)\bar{\phi}_i(g),$$

for all $m \in M$, $g \in SL(2, K_i)$, and $1 \leqq i \leqq l$. When the hypotheses of this theorem are satisfied, and in addition the modules M and $\bar{M}$ are faithful G- and $\bar{G}$-modules, it follows that $G \cong \bar{G}$, and that the modules M and $\bar{M}$ are isomorphic as ΩG-modules.

The second main theorem again concerns finite groups G and $\bar{G}$ satisfying the axioms, and generated by the same number of homomorphic images of $SL(2, K)$, for a given field K. It is also assumed that the p-Sylow subgroups X and $\bar{X}$ of G and $\bar{G}$ respectively, are isomorphic and satisfy a further condition. It is then proved that both G and $\bar{G}$ satisfy the conditions (1)–(13) of Steinberg's paper [12], and consequently possess irreducible modules over Ω of dimension p^M, where p^M is the order of X. Finally it is shown that if neither G nor $\bar{G}$ has a nontrivial center, then the result of the preceding paragraph can be applied to show that G and $\bar{G}$ are isomorphic. The sufficient condition that $G \cong \bar{G}$ involves only group-theoretic properties of G and $\bar{G}$, and no information about modules over G and $\bar{G}$ is needed in order to apply the theorem.

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