A NOTE ON ABSTRACT (M)-SPACES

BY

ANTHONY L. PERESSINI

The following result is a consequence of the theorem that is proved in this note: Every Banach lattice with a strong order unit can be renormed so that the resulting space is an abstract (M)-space with a unit element. As will be seen from the proof, this rather unexpected result is a simple consequence of several known theorems to be found in various places in [2], [3], and [4].

A *locally convex lattice* $E(\mathfrak{T})$ is a vector lattice E over the real field equipped with a Hausdorff locally convex topology $\mathfrak{T}$ which has a generating family $\{p_\alpha\}_{\alpha \in A}$ of semi-norms satisfying

(1) If $|x| \leqq |y|$, then $p_\alpha(x) \leqq p_\alpha(y)$ for all $\alpha \in A$.

A real vector lattice which is a Banach space whose norm satisfies (1) is called a *Banach lattice*. An *abstract* (M)*-space* is a Banach lattice whose norm also satisfies[1]

(2) If $x \geqq \theta$, $y \geqq \theta$, then $\|\sup(x, y)\| = \max\{\|x\|, \|y\|\}$.

A subset H of the positive cone $K = \{x \in E: x \geqq \theta\}$ in a vector lattice E is an *exhausting subset of* K if for each $x \in K$ there are an $h \in H$ and a positive number λ such that $x \leqq \lambda h$. An element $e \in K$ is called a *strong order unit* if $\{e\}$ is an exhausting subset of K. An element $u \in K$ of a Banach lattice E is called a *unit element* if $\|u\| = 1$ and $\|x\| \leqq 1$ implies that $x \leqq u$. More information as well as further references concerning all of the notions defined above, with the exception of that of (M)-space, can be found in [2] and [3]; an account of the basic theory of (M)-spaces is given, for example, in [1].

The properties of the order topology $\mathfrak{T}_0$, introduced independently by Namioka[2] [2] and Schaefer [3], will play a central role in the considerations that follow. $\mathfrak{T}_0$ can be defined as the finest locally convex topology on the vector lattice E for which each order interval

$$[-x, x] = \{z \in E: -x \leqq z \leqq x\} \qquad (x \in K)$$

is a topologically bounded set. Thus a neighborhood basis of the zero element θ is provided by the class of all convex circled sets that absorb each order interval in E. If $E(\mathfrak{T})$ is a locally convex lattice, and if $\{p_\alpha\}_{\alpha \in A}$ is a generating system of semi-norms for $\mathfrak{T}$ satisfying (1), then each p_α is

Received October 13, 1961.

[1] θ denotes the additive identity in E.

[2] Namioka calls $\mathfrak{T}_0$ the "order bound topology $\mathfrak{T}_b$".