ON FINITE GROUPS WITH DIHEDRAL SYLOW 2-SUBGROUPS

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PART I. INTRODUCTION AND PRELIMINARIES

1. Introduction

Much attention has recently been given to the characterization of classes of simple groups in terms of conditions which specify the centralizers of their involutions[2] or their Sylow 2-subgroups. (Cf. R. Brauer [2]; R. Brauer, M. Suzuki, and G. E. Wall [7]; W. Feit [11], M. Suzuki [15], [16], [17], [18], [19]; and J. H. Walter [21], [22].) This paper presents such a characterization for the simple groups[3] $PSL(2, q)$, where q is an odd prime power, and improves the results obtained in [7] and [16].

It is easy to show that in a group with a dihedral[4] Sylow 2-subgroup S the centralizer of an involution τ in the center of S has a normal 2-complement U, and our characterization is given in terms of the structure of U.

THEOREM I. *Let G be a finite group with a dihedral Sylow 2-subgroup S, and let τ be an involution in the center of S. Suppose that the centralizer of τ possesses an abelian 2-complement U. Then G contains a normal subgroup K of odd order and one of the following holds:*

(i) *G has no normal subgroups of index 2, and G/K is isomorphic to $PSL(2, q)$ with q odd or to the alternating group A_7;*

(ii) *G contains a normal subgroup of index 2 but no normal subgroup of index 4, and G/K is isomorphic[3] to $PGL(2, q)$ with q odd;*

(iii) *G contains a normal subgroup of index 4, and G/K is isomorphic to a Sylow 2-subgroup S of G.*

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[2] An involution is understood to be an element of order 2.

[3] These are the groups of unimodular projectivities of a projective line defined with a finite coordinate field F_q of q elements. The groups $PGL(2, q)$ are the groups of projectivities of the projective line defined over F_q. Also $PSL(2, q)$ and $PGL(2, q)$ are the homomorphic images of the unimodular group $SL(2, q)$ and the general linear group $GL(2, q)$ by the homomorphism of $GL(2, q)$ onto $GL(2, q)/Z_2$ where Z_2 is the subgroup of scalar transformations.

[4] We also understand that an elementary abelian group of type (2, 2) is a dihedral group. Such groups will be called *four-groups* in this paper.