BEHAVIOR OF INTEGRAL GROUP REPRESENTATIONS UNDER GROUND RING EXTENSION

BY

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1. Let K be an algebraic number field, and let R be a subring of K containing 1 and having quotient field K. Of primary interest will be the cases

(i) $R = K$,

(ii) R = alg. int. $\{K\}$, the ring of all algebraic integers in K.

(iii) R = valuation ring of a discrete valuation of K.

Given a finite group G, we denote by RG its group ring over R. By an RG-*module* we shall mean a left RG-module which as R-module is finitely generated and torsion-free, and upon which the identity element of G acts as identity operator. Each RG-module M is contained in a uniquely determined smallest KG-module

$$K \otimes_R M,$$

hereafter denoted by KM. For a pair M, N of RG-modules, we write

$$M \sim_R N$$

to denote the fact that $M \cong N$ as RG-modules. The notation

$$M \sim_K N$$

shall mean that $KM \cong KN$ as KG-modules.

Now let K' be an algebraic number field containing K, and let R' be a subring of K' which contains 1 and has quotient field K'. Suppose further that R' is a finitely generated R-module such that

$$R' \cap K = R.$$

Each RG-module M then determines an $R'G$-module denoted by $R'M$, given by

$$R'M = R' \otimes_R M.$$

If M, N are a pair of RG-modules, we write $M \sim_{R'} N$ if $R'M \cong R'N$ as $R'G$-modules. Surely

$$M \sim_R N \Rightarrow M \sim_{R'} N.$$

The reverse implication is false, as we shall see. We propose to investigate more closely the connection between R- and R'-equivalence.

As a first step we may quote without proof a well-known result [9, page 70] which is a consequence of the Krull-Schmidt theorem for KG-modules.

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