ON A THEOREM OF E. H. BROWN

BY

V. K. A. M. GUGENHEIM[1]

Introduction

Recently, E. H. Brown proved that the homology of a fibre space can be computed from a chain complex which is the tensor product of the chains of the base and those of the fibre, the differential operator being a relatively simple modification of the one for the corresponding product space; cf. [6].

Brown proves his theorem for fibre spaces in the sense of Hurewicz, and uses the associated techniques.

A recent paper, [5], shows that, for all purposes of homotopy theory, the chain complex of any fibre space in the sense of Serre can be replaced by a "twisted cartesian product". Thus it seemed immediately that the context of semisimplicial complexes and twisted cartesian products was a natural one for Brown's theorem; and in fact it was found that both his theorem and the proof he gave for it could be adapted to this context. The present paper is devoted to this adaptation.

The following remark may be of interest: The existence of *some* differential for the homology of the total space on the tensor product of base chains by fibre chains follows almost immediately from an (unpublished) lemma of H. Cartan, the proof of which is simple. By its very generality, however, this lemma does not seem to enable one to determine the very special form of the differential given by Brown.

The last section of this paper gives a somewhat generalized version of a theorem of Hurewicz and Fadell [7]; this theorem can be proved more directly in the present context by using the map f of Eilenberg and Mac Lane (cf. Section 5 below) for a direct comparison of the spectral sequence of the tensor product (with the "usual" differential) and the *twisted* cartesian product.

The formulation of the algebraic material in Section 2, in particular the use of the differential operator in Hom (A, C), follows a suggestion of J. C. Moore.

I am greatly indebted to E. H. Brown for letting me see a manuscript of his paper.

1. Algebraic preliminaries

We fix, once and for all, a commutative ring Λ with unit; "module" will mean "Λ-module". If A, B are modules, we write $A \otimes B$, Hom (A, B) for $A \otimes_\Lambda B$, $\mathrm{Hom}_\Lambda(A, B)$.

Received April 3, 1959.

[1] The author was partially supported by the Department of the Army, Office of Ordnance Research.