THE IRREDUCIBILITY OF THE REGULAR SERIES ON AN ALGEBRAIC VARIETY

BY

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Let V be a projective algebraic variety over an algebraically closed field k which will serve as the field of definition for all that follows. There is canonically associated with V a rational mapping $f: V \to A$ of V into an abelian variety A, the Albanese variety of V. The Albanese variety may be defined by the universal mapping property: any rational map $g: V \to B$ of V into an abelian variety B factors as $g = h \circ f$, where h is a rational map of A into B. Classically, A is the torus formed from the period matrix of the q integrals of the first kind on V; when V is a curve, A is just its Jacobian.

It is convenient in what follows to assume that the canonical map f is single-valued; if this is not so to begin with, it will be if we replace V by the graph of the map f on $V \times A$, it being of course birationally equivalent to V. The map f then extends naturally to the set of positive zero-cycles on V by defining $F(x_1 + \cdots + x_n) = \sum f(x_i)$, where the addition on the right refers to the group law on A. We introduce now the n-fold symmetric product $V(n)$ of V with itself: it is definable as the Chow variety which parametrizes all positive zero-cycles of degree n on V. Then F may be viewed as a map $F: V(n) \to A$, which will be single-valued if f is. Such a single-valued, surjective map will be referred to in the sequel as a *foliation*, and the set-theoretic inverse images $F^{-1}(a)$ on V as the *leaves* of the foliation.

The leaves $F^{-1}(a)$ on $V(n)$ represent the equivalence systems of positive zero-cycles of degree n under the natural equivalence relation defined by the mapping F; Albanese called these the "regular series".[2] When V is a curve, the equivalence relation is just linear equivalence. For a study of equivalence relations on zero-cycles of V, it is important to know whether or not these leaves are irreducible varieties, and it is the purpose of this note to show that when n is sufficiently large, this is indeed so. The result we shall prove is the following.

THEOREM. *Let* $\dim V = r > 1$, *let* $q = \dim A$, *and let* g *be the genus of a generic curve on a normal model of* V *(so that* $g \geqq q$*).*

1. *When* $n \geqq g$, *the generic leaf of the (surjective) foliation* $F: V(n) \to A$ *is absolutely irreducible.*

If we let n_0 *be the smallest value of* n *for which this occurs (so that certainly* $n_0 \leqq g$*),*

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[2] G. ALBANESE, *Corrispondenze algebriche fra i punti di due superficie algebriche, I*, Ann. Scuola Norm. Sup. Pisa, ser. II, vol. 3 (1934), p. 1.