LOCAL A-SETS, B-SETS, AND RETRACTIONS[1]

BY

CHRISTOPH J. NEUGEBAUER

Introduction

In the papers [2; 3], L. Cesari introduced the concept of a fine-cyclic element of a mapping (T, J) from a closed finitely connected Jordan region J into the Euclidean space E_3. Fine-cyclic elements constitute a decomposition of proper cyclic elements, and, in case J is unicoherent, coincide with proper cyclic elements. In [5] Cesari's concept of a fine-cyclic element has been extended to a Peano space in the following manner. First, a B-set of a Peano space P has been introduced as a generalization of an A-set of P. Specifically, a B-set B of P is a nondegenerate (more than one point) continuum of P such that either $B = P$ or else each component of $P - B$ has a finite frontier. A fine-cyclic element of P is defined to be a B-set of P whose connection is not destroyed by removing any finite set. It has been shown in [5] that in Peano spaces of finite degree of multicoherence the properties of B-sets and fine-cyclic elements are suitable extensions of the corresponding properties of A-sets and proper cyclic elements.

The first part of this paper shows that fine-cyclic elements are proper cyclic elements relative to some decomposition of a Peano space into a finite number of B-sets.

The second part deals with questions of retractions onto B-sets of a Peano space P. For technical reasons, the concept of a *local A-set* of a Peano space is introduced. For this preliminary survey it suffices to consider a local A-set as a set B which is an A-set relative to some connected open set $G \supset B$. A natural retraction from G onto B suggests itself, namely the one that sends each component of $G - B$ into its frontier relative to G. This retraction is similar to the one used by L. Cesari in [2; 3]. One of the main results of this paper states that this retraction can be extended to P so as to map $P - G$ into a dendrite in B. The last theorem provides some useful information on the composition of two retractions.

It will be shown that every local A-set is a B-set, and in case the underlying Peano space is of finite degree of multicoherence, every B-set is a local A-set.

1. Notation

Let X be a metric space, and let E be a subset of X. The distance function in X will be denoted by ρ, and the diameter of E will be abbreviated by $\delta(E)$. The closure and frontier of E will be designated by $c(E)$ and $\mathrm{Fr}(E)$. If

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