METABELIAN p-GROUPS WITH FIVE GENERATORS AND ORDERS p^{12} AND p^{11}

In commemoration of G. A. Miller

BY

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1. Introduction

This paper continues the study of metabelian groups with elements of order p which are generated by five elements, and which are not direct products of abelian groups and metabelian groups with fewer generators. The problem is stated precisely and the method of investigation is explained in an earlier paper.[1] In that paper the existence and the distinctness of eighty-five such groups of orders from p^{15} to p^{11} were established. This paper will establish the completeness[2] of the list for these orders.

The considerations will all be geometric; nevertheless this is a paper about groups. The groups motivate the study of the complicated considerations required to determine invariants and to show in each case that a given set of invariants is sufficient to characterize a space. We shall be interested in planes and three-spaces in the finite nine-dimensional projective space S which is determined by the Plücker coordinates of the lines of a projective four-space X over GF(p). We classify planes and three-spaces of S under collineations of X.

2. Geometric formulation

We state the problem in geometric terms; the reader is referred to the earlier paper for consideration of the bearing of this study, and also for any proofs required for statements in this section.

Denote the five elements which generate G, any one of these groups, by $U_1, U_2, \cdots, U_5$. Designate commutators of pairs of U's as follows:

$$\begin{array}{lll}
U_2^{-1}U_1U_2 = U_1s_1, & U_3^{-1}U_2U_3 = U_2s_5, & U_4^{-1}U_3U_4 = U_3s_8, \\
U_3^{-1}U_1U_3 = U_1s_2, & U_4^{-1}U_2U_4 = U_2s_6, & U_5^{-1}U_3U_5 = U_3s_9, \\
U_4^{-1}U_1U_4 = U_1s_3, & U_5^{-1}U_2U_5 = U_2s_7, & \\
U_5^{-1}U_1U_5 = U_1s_4, & & U_5^{-1}U_4U_5 = U_4s_{10}.
\end{array}$$

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[1] *Finite metabelian groups and the lines of a projective four-space*, Amer. J. Math., vol. 73 (1951), pp. 539–555.

[2] Strictly, the paper establishes the completeness of a corrected list. Four groups, those connected with spaces of 9′, 20′, 20″, and 21′, were overlooked in the earlier paper. Spaces 20′ and 21′ were first noted by Dr. W. E. Koss and Mr. Peter Yff respectively.