A UNIQUENESS THEOREM FOR MINKOWSKI'S PROBLEM FOR CONVEX SURFACES WITH BOUNDARY

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1. Introduction

By a convex surface with boundary imbedded in a three-dimensional Euclidean space E^3 we mean a two-dimensional compact subset of the boundary of a convex region in the space E^3. The purpose of this paper is to establish the

THEOREM. *Let S and S^* be two orientable convex surfaces of class C^2 with boundaries C and C^*, respectively, and positive Gaussian curvatures imbedded in a three-dimensional Euclidean space E^3. Suppose that there is a differentiable homeomorphism H of the surface S onto the surface S^* such that at corresponding points the two surfaces S and S^* have the same unit inner normal vectors and equal Gaussian curvatures. If the homeomorphism H restricted to the boundary C is a translation carrying the boundary C onto the boundary C^*, then the homeomorphism H is a translation carrying the whole surface S onto the whole surface S^*.*

When the two surfaces S and S^* are closed, the above theorem becomes the well-known uniqueness theorem for Minkowski's problem, which was first established by Minkowski [5] and several decades later proved by Lewy [4] for analytic surfaces S and S^*, by Miranda [6] for surfaces S and S^* having derivatives of 5th order satisfying Hölder's conditions of positive exponent, by Stoker [7] for surfaces S and S^* of class C^3 and by Chern [1] for surfaces S and S^* of class C^2. The methods used by these authors are all different. Minkowski applied Brun-Minkowski's theory concerning the mixed volumes of convex bodies; Lewy, Miranda, and Stoker used different results about elliptic differential equations; and Chern modified the proof of Herglotz [2] for Cohn-Vossen's theorem on isometries of closed convex surfaces by deriving some integral formulas.

The method used in this paper is essentially the same as that used by Chern. It should also be remarked that the uniqueness theorem for Christoffel's problem can also be extended to the same form as that of the above theorem (for this, see [3]).

2. Preliminaries

In a three-dimensional Euclidean space E^3, let us consider a fixed right-handed orthogonal frame $Oe_1 e_2 e_3$, where e_1, e_2, and e_3 form an ordered triple set of mutually orthogonal unit vectors at a point O. Then the position vector of a point P of a surface S of class C^2 in the space E^3 with respect to

Received March 8, 1957.