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THE YAMABE PROBLEM ON MANIFOLDS WITH BOUNDARY: EXISTENCE AND COMPACTNESS RESULTS

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Contents

§0. Introduction. Let (M, g) be an n-dimensional, compact, smooth, Riemannian manifold without boundary. For $n = 2$, we know from the uniformization theorem of Poincaré that there exist metrics that are pointwise conformal to g and have constant Gauss curvature. For $n \geq 3$, the well-known Yamabe conjecture states that there exist metrics that are pointwise conformal to g and have constant scalar curvature. The answer to the Yamabe conjecture is proved to be affirmative through the work of Yamabe [39], Trudinger [38], Aubin [1], and Schoen [31]. See Lee and Parker [23] for a survey. See also Bahri and Brezis [3] and Bahri [2] for works on the Yamabe problem and related ones. For $n \geq 3$, let $\tilde{g} = u^{4/(n-2)} g$ for some positive function $u > 0$ on M; the scalar curvature $R_{\tilde{g}}$ of $\tilde{g}$ can be calculated as

$$R_{\tilde{g}} = u^{-((n+2)/(n-2))} \left(R_g u - \frac{4(n-1)}{n-2} \Delta_g u \right),$$

where R_g denotes the scalar curvature of g. Therefore, the Yamabe conjecture is equivalent to the existence of a solution to

$$-L_g u = \overline{R} u^{(n+2)/(n-2)}, \quad u > 0, \text{in } M, \tag{0.1}$$

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