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CHARACTERISTIC CYCLES FOR THE LOOP GRASSMANNIAN AND NILPOTENT ORBITS

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0. Introduction. Let $\mathcal{F}$ be a perverse sheaf on a complex manifold X, constructible with respect to a Whitney stratification $\mathcal{S}$. A basic problem in microlocal geometry is to compute the characteristic cycle

$$CC(\mathcal{F}) = \sum_{\alpha\in\mathcal{S}} c_\alpha(\mathcal{F})\cdot\overline{T^*_\alpha(X)},$$

which is a linear combination of closures of conormal bundles to submanifolds of X. Intuitively, the microlocal multiplicities $c_\alpha(\mathcal{F})$ measure the singularity of $\mathcal{F}$ at α. In settings related to representation theory, a group G acts on X, $\mathcal{F}$ is G-equivariant, and the microlocal multiplicities play a significant but only partially understood role in representation theory (see [Ro], [SV], [ABV], and [KaSa], e.g.).

In this paper, we compute microlocal multiplicities for certain cases of interest in representation theory. Let G be a connected reductive group with loop group LG, and let P be the subgroup of LG consisting of loops with positive Fourier coefficients. Then P-orbits $\mathcal{G}_\lambda$ on the loop Grassmannian $LG/P = \mathcal{G}$ correspond to the irreducible representations $L(\lambda)$ of the dual reductive group $\check{G}$. For dominant weights μ and λ of a torus of $\check{G}$, let $m_\mu(L(\lambda))$ denote the multiplicity of μ in $L(\lambda)$. Let us embed the orbit closure $\overline{\mathcal{G}_\lambda}$ into a finite-dimensional manifold Z.

THEOREM 0.1. (a) *Let $i:\mathcal{G}_\lambda\to Z$ be the inclusion, and let $i_!(\mathbb{C}_\lambda)$ be the extension by zero of the constant sheaf on $\mathbb{O}_\lambda$. Then $CC(i_!(\mathbb{C}_\lambda)[\dim\mathcal{G}_\lambda]) = \overline{T^*_{\mathcal{G}_\lambda}Z}$.*

(b) *The characteristic cycle of the intersection cohomology sheaf of $\overline{\mathcal{G}_\lambda}$ is given by the character of $L(\lambda)$,*

$$CC(IC_\lambda) = \sum_{\mathcal{G}_\mu\subseteq\overline{\mathcal{G}_\lambda}} m_\mu\big(L(\lambda)\big)\cdot\overline{T^*_{\mathcal{G}_\mu}Z}.$$

We also consider the nilpotent cone in a semisimple Lie algebra.

THEOREM 0.2. *Let $\mathfrak{g}=\mathfrak{sl}(n)$, and let α and β be distinct nilpotent orbits and $i:\alpha\hookrightarrow\mathfrak{g}$. Then the following are true.*

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