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HULLS AND POSITIVE CLOSED CURRENTS

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0. Introduction. A classical problem in complex analysis is describing the polynomial (resp., rational) hull of compact sets in $\mathbb{C}^n$. Recall that a point $x \in \mathbb{C}^n$ is in the polynomial hull $\hat{X}$ of X if and only if $|f(x)| \leqslant \sup_{z \in X} |f(z)|$ for every holomorphic polynomial f. A point x is in the rational convex hull $r(X)$ of X if every algebraic hypersurface through x intersects X.

In a previous paper [DS], we showed that $\hat{X}$ can be described as the union of the supports of positive currents T of bidimension $(1,1)$, compactly supported such that $dd^c T \leqslant 0$ on $\mathbb{C}^n \backslash X$. When S is a totally real compact submanifold, then $r(S) \neq S$ if and only if there is a nonzero compactly supported positive current T of bidimension $(1,1)$ and a current σ of dimension 2 on S such that $dd^c T = dd^c j_* \sigma$, where $j : S \to \mathbb{C}^n$ denotes the inclusion map.

If S is a totally real set, let $\mathscr{C}_{1,1}(S)$ denote the set of nonzero compactly supported positive currents of bidimension $(1,1)$ such that $dT = j_* V$, where V is a current of dimension 1 on S. In [Du], the first author constructed an example of a totally real disc S, which is not polynomially convex although $\mathscr{C}_{1,1}(S)$ is empty.

For S a closed Lagrangian submanifold in $\mathbb{C}^n$, Gromov [G] proved the existence of a nonconstant holomorphic disc with boundary in S. The hypotheses in Gromov's theorem imply that S is without boundary and $\dim_{\mathbb{R}} S = n$. In this case, $\mathscr{C}_{1,1}(S)$ is not empty, because it contains the current of integration on the holomorphic disc. The analytic discs in Gromov's theory are smooth up to the boundary.

When one deals with totally real submanifolds S, even tori in $\mathbb{C}^2$, the existence of such a holomorphic disc is not ensured. For instance, it follows from results of [O] and Gromov's theory that for generic S with a vanishing Maslov class, there is no analytic disc with boundary on S. Using a variation of an example in [DS], Alexander [A2] gave an explicit example of a totally real torus in $\mathbb{C}^2$ bounding no analytic disc.

Building on Gromov's method, Alexander [A1] showed the following result. For any compact totally real (n-dimensional) submanifold S without boundary in $\mathbb{C}^n$, there exists a nonconstant nearly-smooth holomorphic disc with boundary on S. More precisely, there is a bounded nonconstant holomorphic

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