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LAGRANGIAN INTERSECTIONS, SYMPLECTIC ENERGY, AND AREAS OF HOLOMORPHIC CURVES

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1. Introduction and results. In the present paper, it is shown that a closed embedded Lagrangian submanifold L of a geometrically bounded symplectic manifold intersects its image under a Hamiltonian symplectomorphism at as many or more points as the dimension of the $\mathbf{Z}_2$-homology of L, provided the energy of the symplectomorphism is not large and the intersection is transverse.

Let L be a closed embedded Lagrangian submanifold of a symplectic manifold (P, ω). If P is not compact, the symplectic structure should satisfy some conditions implying its reasonable behaviour at infinity. We assume (P, ω) to be tame (geometrically bounded). It means that there exists an almost complex structure J on P such that $(\cdot,\cdot) = \omega(\cdot, J\cdot)$ is a complete Riemannian metric whose sectional curvature is bounded and whose injectivity radius is bounded away from zero; then (P, ω, J) is called a tame almost Kähler manifold (cf. [8], [2]). The basic examples of tame symplectic manifolds are the following: compact symplectic manifolds, cotangent bundles of compact manifolds (equipped with the canonical form $dp \wedge dq$), and symplectic vector spaces $\mathbf{R}^{2n}$.

Denote by $\mathscr{H}(P)$ the space of compactly supported functions on $[0,1] \times P$. Any $H \in \mathscr{H}(P)$ defines a time-dependent Hamiltonian flow on P. Time-1 maps of such flows form a group $\mathscr{G}(P, \omega)$, called the group of exact (or Hamiltonian) symplectomorphisms of P.

Let L transversely meet $g(L)$, where $g \in \mathscr{G}(P, \omega)$. Arnold conjectured that, for some (P, ω) and L, the intersection $L \cap g(L)$ contains at least as many points as the dimension of the homology of L (see [1]). The Lagrangian manifold L is called exact if

$$\int_{D^2} v^*\omega = 0$$

for any $v\colon (D^2, \partial D^2) \to (P, L)$. Floer proved the Arnold conjecture for exact Lagrangian manifolds.

Floer Lagrangian intersection theorem [4] and [7]. *If L is a closed exact Lagrangian submanifold of (P, ω) and $g \in \mathscr{G}(P, \omega)$, then $\#(L \cap g(L)) \geqslant \dim H_*(L, \mathbf{Z}_2)$ provided the intersection is transverse.*

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