

# PROJECTIVE REGULAR MODELS FOR ABELIAN VARIETIES, SEMISTABLE REDUCTION, AND THE HEIGHT PAIRING

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## CONTENTS

0. Introduction .....	161
1. Preliminaries .....	163
2. Degeneration of abelian varieties .....	171
3. Construction of projective regular models in the split complete local case ..	180
4. Semistable reduction and descent .....	191
5. Cycles on singular schemes .....	195
6. The height pairing between algebraic cycles .....	204

**0. Introduction.** Let  $K$  be a number field, and let  $X_\eta$  be a smooth, projective variety over  $\eta = \text{Spec } K$ . In order to describe the behavior of motivic  $L$ -functions associated with the variety  $X_\eta$  near the central point, Beilinson and Bloch defined real-valued height pairings between Chow groups of homologically trivial cycles on  $X_\eta$  that extend the classical pairing of Néron and Tate. Therefore, they have to assume that  $X_\eta$  has a regular model  $X$ , which is proper over the ring of integers in  $K$ , and that homologically trivial cycles on  $X_\eta$  admit suitable extensions to this model. In this paper, we investigate these assumptions in the case of abelian varieties. We construct projective regular models for abelian varieties with semiabelian reduction and prove that they have potentially semistable reduction. Using a nonarchimedean analogue of the  $dd^c$ -lemma for differential forms, we show that there is a well-defined height pairing if an abelian variety has totally degenerate reduction at all places of bad reduction.

In the first part of the paper, we consider the question of finding proper regular models for abelian varieties. Besides the context mentioned above, this question is also raised in [JM]. We give a positive answer to this problem in the case of semiabelian reduction. More precisely, let  $R$  be a Dedekind domain with quotient field  $K$ , and let  $A_\eta$  be an abelian variety over  $\eta = \text{Spec } K$  that admits semiabelian reduction. We use the methods developed by Faltings and Chai in [FC] and Mumford in [M2] to construct a regular model  $P$  of  $A_\eta$  that is projective and flat over  $\text{Spec } R$ . The model  $P$  is by no means unique. Its construction depends on the choice of certain admissible cone decompositions. The reduced

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