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SYMPLECTIC STRUCTURES AND VOLUME ELEMENTS IN THE FUNCTION SPACE FOR THE CUBIC SCHRÖDINGER EQUATION

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CONTENTS

1. Introduction. We consider the cubic Nonlinear Schrödinger (NLS) equation

$$i\psi^{\bullet} = -\psi'' + 2|\psi|^2\psi$$

with periodic boundary conditions. It has an infinite series of conserved quantities —integrals of motion $H_1, H_2, \ldots$. The first three are "classical" integrals:

$$H_1 = \frac{1}{2}\int |\psi|^2 dx = \mathcal{N} = \textit{number of particles},$$

$$H_2 = \frac{1}{2i}\int \psi'\bar{\psi}\, dx = \mathscr{P} = \textit{momentum},$$

$$H_3 = \frac{1}{2}\int |\psi'|^2 + |\psi|^4\, dx = \mathscr{H} = \textit{energy}.$$

The others, $H_4, H_5, \ldots$, do not have classical names.

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