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SUBALGEBRAS OF INFINITE C^*-ALGEBRAS WITH FINITE WATATANI INDICES, II: CUNTZ-KRIEGER ALGEBRAS

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1. Introduction. The index theory of operator algebras was initiated by V. Jones [J] for II_1 factors and later generalized to other classes of operator algebras [K], [PP]. A purely algebraic setting was proposed by Y. Watatani; in [W] he constructed the index theory of C^*-algebras, which has a certain relationship to K-theory, by manipulating Hilbert C^*-modules and quasi bases, which are generalizations of Pimsner-Popa bases [PP]. Further development of the theory was recently achieved by T. Kajiwara and Y. Watatani [KW], where a relationship to KK-theory was pointed out.

In our previous work [I3], we constructed examples of subalgebras of the Cuntz algebras with finite Watatani indices. We started with certain fusion rules of sectors of infinite factors, and obtained endomorphisms of the Cuntz algebras, whose images have finite Watatani indices.

In the present work, we modify our previous construction and apply it to every subfactor; starting from every biunitary connection on every finite graph, we obtain an inclusion of the Cuntz-Krieger algebras with a finite Watatani index. Our construction also works in the infinite-depth case; however, obtained algebras are a priori not necessarily the Cuntz-Krieger algebras. If the graphs are of depth 2 and the connection is flat, our construction gives the Cuntz algebras and pairs of endomorphisms, which give rise to actions of multiplicative unitaries in the sense of J. Cuntz [C2], [L5]. By using these facts, we construct examples of braided endomorphisms of the Cuntz-Krieger algebras (and the AFD type III_λ factor), and, in consequence, of unitary representations of the braid group B_∞ with Markov traces. For this purpose, we introduce the notion of self-conjugate structures of finite-dimensional Kac algebras, and give explicit equations that describe braiding operators.

This paper is organized as follows. In Section 2, we investigate the combinatorial structure of subfactors of infinite factors with finite indices. For this purpose, we reformulate the Frobenius reciprocity for sectors, which has its own interest [cf. O4]. Usually, one obtains a pair of AF algebras through basic construction. However, due to the advantage of infiniteness, we have the canonical endomorphism invented by R. Longo. Therefore, we associate with a pair of

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