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POSITIVE SOLUTIONS OF YAMABE-TYPE EQUATIONS ON THE HEISENBERG GROUP

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1. Introduction and notation. Let H^n be the Heisenberg group of real dimension $2n+1$, that is, the nilpotent Lie group which, as a manifold, is the product

$$H^n = \mathbb{C}^n \times \mathbb{R}$$

and whose group structure is given by

$$(z,t) \circ (z',t') = (z+z', t+t'+2\operatorname{Im}(z,z')), \qquad (z,t),\ (z',t') \in H^n,$$

where $(\ ,\ )$ denotes the usual Hermitian product on $\mathbb{C}^n$.

A (real) basis for the Lie algebra of left-invariant vector fields on H^n is given by

$$X_j = 2\operatorname{Re}\frac{\partial}{\partial z_j} + 2\operatorname{Im} z_j \frac{\partial}{\partial t}, \quad Y_j = 2\operatorname{Im}\frac{\partial}{\partial z_j} - 2\operatorname{Re} z_j \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial t} \tag{1.1}$$

for $j = 1, 2, \ldots, n$. The above basis satisfies Heisenberg's canonical commutation relations for position and momentum,

$$[X_j, Y_k] = -4\delta_{jk}\frac{\partial}{\partial t}, \tag{1.2}$$

all other commutators being zero. It follows that the vector fields X_j, Y_k satisfy Hörmander's condition, and the real part of the Kohn-Spencer Laplacian, defined as

$$\Delta_{H^n} = \sum_{j=1}^{n}(X_j^2 + Y_j^2), \tag{1.3}$$

is hypoelliptic by Hörmander's theorem (see [11]).

In H^n one has a natural origin $0 = (0,0)$ and a distinguished distance function from zero defined by

$$d(x) = d(z,t) = (|z|^4 + t^2)^{1/4}, \tag{1.4}$$

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